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Semi-inclusive polarized DIS in terms of Mellin moments.
I. Light sea quark polarized distributions [1].

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Abstract

In connection with the semi-inclusive polarized DIS, it is proposed to consider the first Mellin moments Δq of the polarized quark and antiquark densities, instead of the respective variables $\delta q(x)$, local in Bjorken x themselves. This gives rise to a very essential simplification of the next to leading order (NLO) QCD and, besides, allows one to use the respective QCD sum rules. An expression for $\Delta\bar{u} - \Delta\bar{d}$ in NLO is obtained which is just a simple combination of the directly measured asymmetries and of the quantities taken from the unpolarized data.

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Investigation of the quark structure of the nucleon is one of most important tasks of modern high energy physics. In this respect deep inelastic scattering (DIS) is of special importance. Thus, the very impressive result of the New Muon Collaboration (NMC) experiment was obtained in 1991, when the unpolarized structure functions of the proton and neutron, $F_2^p(x)$ and $F_2^n(x)$, were precisely measured within a wide range of Bjorken's x, and, it was established that the integral $\int_0^1 \frac{dx}{x}[F_2^p(x) - F_2^n(x)]$ does not equal 1/3 (Gottfried sum rule) but has a much smaller value 0.235 ± 0.0026. This means that the densities of u and d sea quarks, $\bar{u}(x)$ and $\bar{d}(x)$, in the proton have different values, and

$$\int_0^1 dx\ [\bar{d}(x) - \bar{u}(x)] = 0.147 \pm 0.039 \neq 0.$$

In polarized DIS, instead of the unpolarized total $q = q^{\uparrow} + q_{\downarrow}$, sea $\bar{q}$ and valence $q_V = q - \bar{q}$ quark densities, the set of the respective polarized quantities $\delta q(x, Q^2) = q^{\uparrow}(x, Q^2) - q_{\downarrow}(x, Q^2)$, $\delta\bar{q}(x, Q^2) = \bar{q}^{\uparrow}(x, Q^2) - \bar{q}_{\downarrow}(x, Q^2)$ and $\delta q_V(x, Q^2) = \delta q(x, Q^2) - \delta\bar{q}(x, Q^2)$ is the subject of the investigation. So, the question arises: does the difference between the polarized u and d sea quark densities $\delta\bar{u}(x, Q^2) - \delta\bar{d}(x, Q^2)$ also differ from zero? Recently, a series of theoretical papers appeared ([1-4]) where it was predicted that the quantity $\delta\bar{u}(x, Q^2) - \delta\bar{d}(x, Q^2)$ does not equal zero. However, the model-dependent results for $\delta\bar{u}(x, Q^2) - \delta\bar{d}(x, Q^2)$ essentially differ each from other in these papers. So, it is very desirable to find a reliable way to extract this quantity directly from experiment data. For this purpose it is not sufficient to use just the inclusive polarized DIS data, and one has to investigate semi-inclusive polarized DIS processes like

$$\vec{\mu} + \vec{p}(\vec{d}) \to \mu + h + X.$$

Such processes provide direct access to the individual polarized quark and antiquark distributions via measurements of the respective *spin asymmetries.* [3]

Unfortunately, the description of semi-inclusive DIS processes turns out to be much more complicated in comparison with the traditional inclusive polarized DIS. First, the fragmentation functions are involved, for which no quite reliable information is available[4]. Second (and this is the most serious problem), the consideration even of the next to leading (NLO) QCD order turns out to be extremely difficult, since it involves double convolution products. So, to achieve a reliable description it is very desirable, on the one hand, to exclude from consideration the fragmentation functions, whenever possible, and, on the other hand (and this is the main task), to try to simplify the NLO consideration as much as possible, without which one can say nothing about the reliability and stability of results obtained within the quark-parton model (QPM).

It is well known that within QPM one can completely exclude the fragmentation functions from the expressions for the valence quark polarized distributions δq_V through experimentally measured asymmetries. To this end, instead of the usual virtual photon asymmetry $A^h_{\gamma N} \equiv A^h_{1N}$ (which is expressed in terms of the directly measured asymmetry $A^h_{exp} = (n^h_{\uparrow\downarrow} - n^h_{\uparrow\uparrow})/(n^h_{\uparrow\downarrow} + n^h_{\uparrow\uparrow})$ as $A^h_{1N} = (P_B P_T f D)^{-1} A^h_{exp}$), one has to measure so called "difference asymmetry" $A_N^{h^+-h^-}$ [6] (see also [5,7]) which is expressed in terms of the

[3] Such a kind of measurements were performed by SMC and HERMES experiments and are also planned by the COMPASS collaboration.

[4] For discussion of this subject see, for example [5] and references therein.

respective counting rates as

$$A_N^{h-\bar h}(x,Q^2;z)=\frac{1}{P_BP_TfD}\frac{(n^h_{\uparrow\downarrow}-n^{\bar h}_{\uparrow\downarrow})-(n^h_{\uparrow\uparrow}-n^{\bar h}_{\uparrow\uparrow})}{(n^h_{\uparrow\downarrow}-n^{\bar h}_{\uparrow\downarrow})+(n^h_{\uparrow\uparrow}-n^{\bar h}_{\uparrow\uparrow})}, \tag{1}$$

where the event densities $n^h_{\uparrow\downarrow(\uparrow\uparrow)}=dN^h_{\uparrow\downarrow(\uparrow\uparrow)}/dz$, i.e. $n^h_{\uparrow\downarrow(\uparrow\uparrow)}dz$ are the numbers of events for antiparallel (parallel) orientations of here muon and target nuclear (proton or deutron here) spins for the hadrons of type h registered in the interval dz. Coefficients P_B and P_T, f and D are the beam and target polarizations, dilution and depolarization factors, respectively,(for details on these coefficients see, for example, [8,9] and references therein). Then, the QPM expressions for the difference asymmetries look like (see, for example, COMPASS project [10], appendix A)

$$\begin{aligned}
A_p^{\pi^+-\pi^-} &= \frac{4\delta u_V-\delta d_V}{4u_V-d_V}; \quad A_n^{\pi^+-\pi^-}=\frac{4\delta d_V-\delta u_V}{4d_V-u_V};\\
A_d^{\pi^+-\pi^-} &= \frac{\delta u_V+\delta d_V}{u_V+d_V};\\
A_p^{K^+-K^-} &= \frac{\delta u_V}{u_V};\\
A_d^{K^+-K^-} &= A_d^{\pi^+-\pi^-},
\end{aligned} \tag{2}$$

i.e., on the one hand, they contain only valence quark polarized densities, and, on the other hand, have the remarkable property to be free of any fragmentation functions.

All this is very good, but we are interested here in the sea quark polarized distributions, and, besides, the main question arises - what will happen with all this beauty in the next to leading order QCD?

We propose to investigate the *integral* quantities, namely, the first Mellin moments $M^1(\delta q)\equiv\int_0^1 dx\,[\delta q(x)]\equiv\Delta q$ $(q=u,d,s,...)$ instead of the local polarized quark densities $\delta q(x)$ themselves. This provides very essential *advantages:*

First.

Even if the local quantity has a very small [5] value at each point x, the integral of this quantity over the whole range of x-variables may already have quite a considerable value, and, one can hope that QPM turns out to be a good approximation for integral quantities like

$$\Delta\bar u-\Delta\bar d\equiv\int_0^1 dx\,[\delta\bar u(x)-\delta\bar d(x)]. \tag{3}$$

An argument in favor of such a hope (for (3)) is the circumstance that all the model predictions [1-4] have one common feature: the local quantity $\delta\bar u(x)-\delta\bar d(x)$ does not change sign when x varies over its entire range $0\le x\le 1$.

Second.

To investigate integral quantities like (3) one can use *QCD sum rules.* In particular, one can apply such a well established sum rule as *the Bjorken sum rule*[6]

$$\int_0^1 dx[g_1^p-g_1^n]=\frac{1}{6}\,\frac{g_A}{g_V}(1-\frac{\alpha_s(Q^2)}{\pi}+O(\alpha_s^2)), \tag{4}$$

[5] Notice, however, that the latest theoretical paper [4] on this subject predicts that the difference between the polarized densities $\delta\bar u$ and $\delta\bar d$ should be even more significant than the difference between the unpolarized sea quark densities: $|\delta\bar u-\delta\bar d|\ge|\bar u-\bar d|$.

[6] Throughout the paper, all the quantities considered in NLO are given in the $\overline{\mathrm{MS}}$ scheme.

$$g_A/g_V = 1.2537 \pm 0.0028$$

to express the quantity $\Delta\bar{u} - \Delta\bar{d}$ of interest via the quantity $\Delta u_V - \Delta d_V$ which, in turn, is expressed via the measured difference asymmetries $A_p^{\pi^+-\pi^-}$ and $A_d^{\pi^+-\pi^-}$.

Third (and we consider this *the most important advantage* of the proposed procedure) Application of the Mellin moments, instead of the local quantities themselves, results in a remarkable simplification of the NLO QCD consideration of the semi-inclusive polarized DIS, that is extremely complicated in terms of the local quantities.

Thus, let us consider the NLO [11] expression for the structure function g_1^p

$$g_1^p(x, Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2 \left(\delta q + \frac{\alpha_s(Q^2)}{2\pi}[C_q \otimes \delta q + C_g \otimes \delta g]\right)(x, Q^2), \tag{5}$$

where

$$\Big(C \otimes f\Big)(x) \equiv \int_x^1 \frac{dy}{y}\, C\left(\frac{x}{y}\right) f(y) \tag{6}$$

is the definition of the convolution product. From now on we will use the well known remarkable property of the Mellin n-th moments

$$M^n(f) \equiv \int_0^1 dx\; x^{n-1} f(x) \tag{7}$$

to split the convolution product (6) into a simple product of the Mellin moments of the respective functions:

$$M^n[C \otimes f] \equiv \int_0^1 dx x^{n-1} \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right) f(y) = M^n(C) M^n(f). \tag{8}$$

So, taking the first Mellin moment of Eq. (5) and using the expressions for the Mellin moments of the respective NLO ($\overline{\mathrm{MS}}$) Wilson coefficients

$$M^1(C_q) = -2, \quad M^1(C_g) = 0$$

one obtains [11] in NLO QCD:

$$M^1[g_1^p] \equiv \int_0^1 dx\; g_1^p(x, Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2 \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) \int_0^1 dx\; \delta q \tag{9}$$

and the same for g_1^n with the substitution $u \leftrightarrow d$. Substituting the last expressions for $M^1[g_1^p]$ and $M^1[g_1^n]$ into the Bjorken sum rule (4), one can see that the α_s dependent multipliers $(1 - \alpha_s(Q^2)/\pi)$ cancel out precisely in the left- and right-hand sides, and, one arrives at the simple relation between the polarized densities of sea and valence quarks

$$\int_0^1 dx\; (\delta\bar{u} - \delta\bar{d}) = \frac{1}{2}\,\frac{g_A}{g_V} - \frac{1}{2}\int_0^1 dx\; (\delta u_V - \delta d_V), \tag{10}$$

or, in the notation used here,

$$\Delta\bar{u} - \Delta\bar{d} = \frac{1}{2}\,\frac{g_A}{g_V} - \frac{1}{2}\;(\Delta u_V - \Delta d_V). \tag{11}$$

Thus, the relation between the first Mellin moments of the polarized sea and valence quark distributions has a very simple form and does not contain α_s dependence at all (i.e. is an exact relation at least up to $O(\alpha_s^2)$ corrections).

With such a simple relation between $\Delta\bar{u} - \Delta\bar{d}$ and $\Delta u_V - \Delta d_V$ at our disposal, *the next step* is to establish the relation between the Mellin moments Δu_V and Δd_V and the experimentally observable difference asymmetries $A_{p(d)}^{\pi^+-\pi^-}$ in NLO QCD. For this purpose, one can use the following relations [5,12,13] for the difference asymmetries

$$A_N^{h-\bar{h}}(x, Q^2; z) = \frac{g_1^{N/h} - g_1^{N/\bar{h}}}{\tilde{F}_1^{N/h} - \tilde{F}_1^{N/\bar{h}}} \quad (N = p, n, d), \tag{12}$$

where the semi-inclusive analogs of the structure functions g_1^N and F_1^N, functions $g_1^{N/h}$ and $\tilde{F}_1^{N/h}$, are related to the respective polarized and unpolarized semi-inclusive differential cross-sections as follows [12]

$$\frac{d^3\sigma^h_{N\uparrow\downarrow}}{dxdydz} - \frac{d^3\sigma^h_{\uparrow\uparrow}}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}\,(2-y)\; g_1^{N/h}(x, z, Q^2), \tag{13}$$

$$\frac{d^3\sigma^h_{N}}{dxdydz} = \frac{2\pi\alpha^2}{Q^2}\,\frac{1+(1-y)^2}{y}\; 2\tilde{F}_1^{N/h}(x, z, Q^2), \tag{14}$$

where

$$\tilde{F}_1^{N/h} \equiv F_1^{N/h} + \frac{1-y}{1+(1-y)^2}\; F_L^{N/h}. \tag{15}$$

The semi-inclusive structure functions $g_1^{p(n)/h}$ are given in NLO by

$$\begin{aligned} 2g_1^{p/h} &= \sum_{q,\bar{q}} e_q^2 \delta q[1 + \otimes\frac{\alpha_s}{2\pi}\delta C_{qq}\otimes] D_q^h \\ &+ (\sum_{q,\bar{q}} e_q^2 \delta q) \otimes \frac{\alpha_s}{2\pi}\delta C_{gq} \otimes D_g^h \\ &+ \delta g \otimes \frac{\alpha_s}{2\pi}\delta C_{qg} \otimes (\sum_{q,\bar{q}} e_q^2 D_q^h), \end{aligned} \tag{16}$$

$$g_1^{n/h} = g_1^{p/h}\Big|_{u\leftrightarrow d,\ s\leftrightarrow s}, \tag{17}$$

where

$$[A\otimes B\otimes C](x, z) \equiv \int_{\mathcal{D}}\!\int \frac{dx'}{x'}\frac{dz'}{z'} A\left(\frac{x}{x'}\right) B(x', z') C\left(\frac{z}{z'}\right) \tag{18}$$

is the double convolution product. The respective expressions for $2\tilde{F}_1^{p(n)/h}$ have the same form with the substitution $\delta q \to q$, $\delta C \to \tilde{C}$. The expressions for the Wilson coefficients $\delta C_{qq(qg,gq)}$ and $\tilde{C}_{qq(qg,gq)} \equiv C^1_{qq(qg,gq)} + 2[(1-y)/(1+(1-y)^2)]C^L_{qq(qg,gq)}$ can be found, for

example, in [12], Appendix C.
It is remarkable that due to the properties of the fragmentation functions:

$$\begin{aligned} D_1 &\equiv D_u^{\pi^+} = D_{\bar u}^{\pi^-} = D_{\bar d}^{\pi^+} = D_d^{\pi^-}, \\ D_2 &\equiv D_d^{\pi^+} = D_{\bar d}^{\pi^-} = D_u^{\pi^-} = D_{\bar u}^{\pi^+}, \end{aligned} \tag{19}$$

in the differences $g_1^{p/\pi^+} - g_1^{p/\pi^-}$ and $\tilde F_1^{p/\pi^+} - \tilde F_1^{p/\pi^-}$ (and, therefore, in the asymmetries $A_p^{\pi^+-\pi^-}$ and $A_d^{\pi^+-\pi^-}$) only the contributions containing the Wilson coefficients δC_{qq} and $\tilde C_{qq}$ survive. However, even then the system of double integral equations

$$\begin{aligned} A_p^{\pi^+-\pi^-}(x,Q^2;z) &= \frac{(4\delta u_V - \delta d_V)[1+\otimes\alpha_s/(2\pi)\delta C_{qq}\otimes](D_1-D_2)}{(4u_V-d_V)[1+\otimes\alpha_s/(2\pi)C_{qq}\otimes](D_1-D_2)}(x,Q^2;z), \\ A_n^{\pi^+-\pi^-}(x,Q^2;z) &= A_p^{\pi^+-\pi^-}(x,Q^2;z)|_{u_V\leftrightarrow d_V} \end{aligned} \tag{20}$$

proposed by E. Christova and E. Leader [5], is extremely difficult to solve with respect to the local quantities $\delta u_V(x,Q^2)$ and $\delta d_V(x,Q^2)$. Besides, the range of integration $\mathcal{D}$ used in ref. [5] has a very complicated form, namely:

$$\frac{x}{x+(1-x)z} \le x' \le 1 \; with \; z \le z' \le 1, \tag{21}$$

if $x+(1-x)z \ge 1$, and, additionally, range

$$x \le x' \le x/(x+(1-x))z$$

with $x(1-x')/(x'(1-x)) \le z' \le 1$ if $x+(1-x)z \le 1$. So, one can see that here even application of the Mellin moments cannot simplify the situation.

Such enormous complification of the convolution integral range occurs if one introduces (to take into account the target fragmentation contributions[7] and to exclude the cross-section singularity problem at $z_h = 0$) a new hadron kinematical variable $z = E_h/E_N(1-x)$ (γp c.m. frame) instead of the usual semi-inclusive variable $z_h = (Ph)/(Pq) = (lab.system)\ E_h/E_\gamma$. However, both problems compelling us to introduce z, instead of z_h, can be avoided (see, for example [12,13]) if one, just to neglect the target fragmentation, applies a proper kinematical cut $Z < z_h \le 1$, i.e. properly restricts the kinematical region covered by the final state hadrons[8]. Then, one can safely use, instead of z, the usual variable z_h, which at once makes the integration range $\mathcal{D}$ in the double convolution product (18) very simple:[9] $x \le x' \le 1,\ z_h \le z' \le 1$. Note that in applying the kinematical cut it is much more convenient to deal with the total numbers of events

$$N^h_{\uparrow\downarrow(\uparrow\uparrow)}(x,Q^2)\Big|_Z = \int_Z^1 dz_h\; n^h_{\uparrow\downarrow(\uparrow\uparrow)}(x,Q^2;z_h) \tag{22}$$

within the entire interval $z \le z_h \le 1$ and the respective integral difference asymmetries [10]

$$A_N^{h-\bar h}(x,Q^2)\Big|_Z = \frac{1}{P_B P_T f D}\frac{(N^h_{\uparrow\downarrow} - N^{\bar h}_{\uparrow\downarrow}) - (N^h_{\uparrow\uparrow} - N^{\bar h}_{\uparrow\uparrow})}{(N^h_{\uparrow\downarrow} - N^{\bar h}_{\uparrow\downarrow}) + (N^h_{\uparrow\uparrow} - N^{\bar h}_{\uparrow\uparrow})}\Big|_Z =$$

[7]Then, one should also add the target fragmentation contributions to the right-hand side of (16).

[8]This is just what was done in the HERMES and COMPASS experiments, where the applied kinematical cut was $z_h > Z = 0.2$.

[9] Namely the such range was used in the early seminal papers [14] (see also [12]).

[10]Namely the integral spin symmetries $A^h_{1N} = \int_Z^1 dz_h\, g_1^{N/h} \Big/ \int_Z^1 dz_h\, \tilde F_1^{N/h}$ were measured by SMC and HERMES experiments (see [8,9] and also [13]).

$$= \frac{\int_Z^1 dz_h (g_1^{N/h} - g_1^{N/\bar{h}})}{\int_Z^1 dz_h (\tilde{F}_1^{N/h} - \tilde{F}_1^{N/\bar{h}})} \quad (N = p, n, d), \tag{23}$$

than with the local in z_h quantities $n_{\uparrow\downarrow(\uparrow\uparrow)}(x, Q^2; z_h)$ and $A_N^{h-\bar{h}}(x, Q^2; z_h)$. So, the expressions for the proton and deutron integral difference asymmetries assume the form[11]

$$A_p^{\pi^+-\pi^-}(x, Q^2)\Big|_Z = \frac{(4\delta u_V - \delta d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \delta C_{qq} \otimes](D_1 - D_2)}{(4u_V - d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2)}, \tag{24}$$

$$\begin{aligned} A_d^{\pi^+-\pi^-}(x, Q^2)\Big|_Z &= \frac{\int_Z^1 dz_h \, [(g_1^{p/\pi^+} - g_1^{p/\pi^-}) + (g_1^{n/\pi^+} - g_1^{n/\pi^-})]}{\int_Z^1 dz_h [(\tilde{F}_1^{p/\pi^+} - \tilde{F}_1^{p/\pi^-}) + (\tilde{F}_1^{n/\pi^+} - \tilde{F}_1^{n/\pi^-})]} = \\ &= \frac{(\delta u_V + \delta d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \delta C_{qq} \otimes](D_1 - D_2)}{(u_V + d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2)}, \end{aligned} \tag{25}$$

and the double convolution reads

$$[A \otimes B \otimes C] = \int_x^1 \frac{dx'}{x'} \int_{z_h}^1 \frac{dz'}{z'} \, A\left(\frac{x}{x'}\right) B(x', z') C\left(\frac{z_h}{z'}\right). \tag{26}$$

Now, application of the first Mellin moment to the difference asymmetries $A_p^{\pi^+-\pi^-}(x, Q^2)|_Z$ and $A_d^{\pi^+-\pi^-}(x, Q^2)|_Z$, given by (24) – (26), becomes extremely useful and allows one to obtain a system of two *purely algebraic* equations for $\Delta u_V \equiv \int_0^1 dx \; \delta u_V$ and $\Delta d_V \equiv \int_0^1 dx \; \delta d_V$:

$$(4\Delta u_V - \Delta d_V)(M_1 - M_2) = \mathcal{A}_p^{exp}, \tag{27}$$

$$(\Delta u_V + \Delta d_V)(M_1 - M_2) = \mathcal{A}_d^{exp}, \tag{28}$$

with the solution

$$\Delta u_V = \frac{1}{5} \frac{\mathcal{A}_p^{exp} + \mathcal{A}_d^{exp}}{M_1 - M_2}; \quad \Delta d_V = \frac{1}{5} \frac{4\mathcal{A}_d^{exp} - \mathcal{A}_p^{exp}}{M_1 - M_2}. \tag{29}$$

Here we introduce the notation

$$\begin{aligned} \mathcal{A}_p^{exp} &\equiv \int_0^1 dx \; A_p^{\pi^+-\pi^-}\Big|_Z (4u_V - d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2), \\ \mathcal{A}_d^{exp} &\equiv \int_0^1 dx \; A_d^{\pi^+-\pi^-}\Big|_Z (u_V + d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2), \end{aligned} \tag{30}$$

and

$$\begin{aligned} M_1 &\equiv M_u^{\pi^+} = M_{\bar{u}}^{\pi^+} = M_{\bar{d}}^{\pi^+} = M_d^{\pi^-}, \\ M_2 &\equiv M_d^{\pi^+} = M_{\bar{d}}^{\pi^-} = M_u^{\pi^-} = M_{\bar{u}}^{\pi^+}, \end{aligned} \tag{31}$$

[11] To obtain (25) one uses that the deutron cross section is the sum of the proton and neutron cross sections, which is valid up to corrections of order $O(\omega_D)$, where $\omega_D = 0.05 \pm 0.01$ is the probability to find deutron in the D-state.

where

$$M_q^h(Q^2) \equiv \int_Z^1 dz_h \left[D_q^h(z_h, Q^2) + \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz'}{z'} \, \Delta C_{qq}(z') D_q^h(\frac{z_h}{z'}, Q^2) \right] \tag{32}$$

with the coefficient

$$\Delta C_{qq}(z) \equiv \int_0^1 dx \; \delta C_{qq}(x,z), \tag{33}$$

that is given in Appendix. Thus, using the relation (11) between $\Delta u - \Delta d$ and $\Delta u_V - \Delta d_V$ one gets, eventually, a simple expression for $\Delta\bar{u} - \Delta\bar{d} \equiv \int_0^1 dx \; (\delta\bar{u}(x,Q^2) - \delta\bar{d}(x,Q^2))$ in terms of experimentally measured quantities, that is valid in NLO QCD :

$$\Delta\bar{u} - \Delta\bar{d} = \frac{1}{2} \, \frac{g_A}{g_V} - \frac{2\mathcal{A}_p^{exp} - 3\mathcal{A}_d^{exp}}{10(M_1 - M_2)}. \tag{34}$$

It is easy to see that all the quantities present in the right-hand side, with the exception of the two difference asymmetries $A_p^{\pi^+-\pi^-}|_Z$ and $A_d^{\pi^+-\pi^-}|_Z$ (entering into $\mathcal{A}_p^{exp}$ and $\mathcal{A}_d^{exp}$, respectively) can be extracted from *unpolarized*[12] semi - inclusive data and can, thus, be considered here as a known input. So, the only quantities that have to be measured in *polarized* semi-inclusive DIS are the difference asymmetries $A_p^{\pi^+-\pi^-}|_Z$ and $A_d^{\pi^+-\pi^-}|_Z$ which, in turn, are just simple combinations of the directly measured counting rates.

In conclusion, we would like to stress that application of the Mellin moments, instead of the local polarized densities, happens to be very fruitful not only in the case of light u- and d-quarks, but also for investigation of polarized strangeness in the nucleon (a paper is now in preparation). Besides, we also plan to apply this procedure to the transverse asymmetries in the nearest future.

At present, a proposal for measurement of $\Delta\bar{u} - \Delta\bar{d}$, based on the above described procedure, is being prepared for the experiment COMPASS in collaboration with the group of INFN – sezione di Torino and of Dipartimento di fisica generale "A.Avogadro" of the Torino University.

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Appendix: Mellin moments of polarized semi-inclusive DIS Wilson coefficients.

The NLO ($\overline{MS}$) coefficient δC_{qq} has the form (see [12], Appendix C)

$$\delta C_{qq} = C_{qq}^1 - 2C_F(1-x)(1-z) \quad (C_F = 4/3), \tag{35}$$

where

[12] With the standard and well established assumption that the fragmentation functions do not depend on the spin. Then, the unpolarized fragmentation functions D can be taken either from independent measurements of e^+e^- - annihilation into hadrons [15] or in hadron production in unpolarized DIS [16]

$$
\begin{aligned}
C^1_{qq} &= C_F\Big\{-8\delta(1-x)\delta(1-z)+\delta(1-x)\Big[\tilde P_{qq}(z)\ln\frac{Q^2}{M_F^2}+L_1(z)+L_2(z)\\
&+ (1-z)\Big]+\delta(1-z)\left[\tilde P_{qq}(x)\ln\frac{Q^2}{M^2}+L_1(x)-L_2(x)+(1-x)\right]+\\
&+ 2\frac{1}{(1-x)_+}\frac{1}{(1-z)_+}-\frac{1+z}{(1-x)_+}-\frac{1+x}{(1-z)_+}+2(1+xz)\Big\},
\end{aligned}
\tag{36}
$$

$$
\begin{gathered}
\tilde P_{qq}(z)=\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z),\\
L_1(z)=(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+,\quad L_2(z)=\frac{1+z^2}{1-z}\ln z,
\end{gathered}
\tag{37}
$$

and the "+" prescription is given by

$$
\int_0^1 dz\ f(z)(g(z))_+\equiv\int_0^1 dz\ [f(z)-f(1)]\ g(z).
\tag{38}
$$

Simple calculation of the n-th moment $M^n(\delta C_{qq})$ gives

$$
\begin{aligned}
M_n(\delta C_{qq}) = C_F\Big\{&\delta(1-z)\Big[-8+\frac{3}{2}\ln\frac{Q^2}{M_F^2}+\ln\frac{Q^2}{M^2}\left(\frac{3}{2}-2\gamma-\frac{1}{n}-\frac{1}{1+n}-2\Psi(n)\right)\\
&+\frac{1}{6}\Big(6\gamma^2+3\left(\frac{1}{n^2}+\frac{1}{(1+n)^2}\right)+6\gamma\left(\frac{1}{n}+\frac{1}{1+n}\right)+\pi^2+12\gamma\Psi(n)+3\Psi^2(n)\\
&+3\Psi^2(n+2)-6\frac{d\Psi(n)}{dn}\Big)+\zeta(2,n)+\zeta(2,2+n)+\frac{1}{n(n+1)}\Big]\\
&-\frac{2}{(1-z)_+}[\gamma+\Psi(n)]+(1+z)[\gamma+\Psi(n)]\\
&-\frac{1}{(1-z)_+}\left(\frac{1}{n}+\frac{1}{1+n}\right)+2\left(\frac{1}{n}+\frac{z}{1+n}\right)\\
&+\tilde P_{qq}(z)\ln\frac{Q^2}{M_F^2}+L_1(z)+L_2(z)+(1-z)\frac{n^2-3n-2}{n(n+1)}\Big\},
\end{aligned}
$$

where $\Psi(z)=\Gamma'(z)/\Gamma(z)$; $\gamma\simeq 0.577216$ is the Euler constant.

For example, the first two moments are

$$
\begin{aligned}
M_1\left(\delta C_{qq}\right)\equiv\Delta C_{qq}=C_F\Big[&1+2z-\frac{3}{2}\frac{1}{(1-z)_+}+\delta(1-z)\left(-7+\frac{\pi^2}{3}+\frac{3}{2}\ln\frac{Q^2}{M_F^2}\right)+\\
&+\tilde P_{qq}(z)\ln\frac{Q^2}{M_F^2}+L_1(z)+L_2(z)\Big];
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
M_2\left(\delta C_{qq}\right)=C_F\Big[&\frac{5}{3}+2z-\frac{17}{6}\frac{1}{(1-z)_+}+\frac{1}{6}\delta(1-z)\Big(-41+2\pi^2-\\
&8\ln\frac{Q^2}{M^2}+9\frac{Q^2}{M_F^2}\Big)+\tilde P_{qq}(z)\ln\frac{Q^2}{M_F^2}+L_1(z)+L_2(z)+\frac{2}{3}(1-z)\Big].
\end{aligned}
\tag{40}
$$

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